Digital surface analysis: – A new approach using differential geometry

1st James K. Dirstein
TotalDepth
21 Churchill Avenue
Subiaco, 6008 WA, Australia
jim@td.iinet.net.au

2nd Pavol Ihring
GeoProxima
Suite 4, 11 Victoria Parade
Manly, 2095 NSW, Australia
pavol.ihring@geoproxima.com.au

3rd Stano Hroncek
GeoProxima
Suite 4, 11 Victoria Parade
Manly, 2095 NSW, Australia
stano.hroncek@geoproxima.com.au

SUMMARY

After more than a decade of research and development our Eureka Moment is presented discussing a new differential geometry solution applied to the problem of digital surface analysis. The solution applies a completely different approach in mathematics without the use of existing techniques or algorithms. The process entails the calculation of a complete set of morphometric properties for the surface as it is defined by Differential Geometry. All processing of the data is automated, fast and accurately locates objects within the surface without the introduction of high frequency artefacts commonly associated with existing approaches. A number of objective evaluation methods are demonstrated offering comparative analysis with other published technologies on known mathematical models (with noise). Real data examples are provided showing the application of this technology on the analysis of data surfaces from seismic and potential fields surveys The queryable database of accurate and high quality elements becomes and essential aspect to highly simplify and speed up the data mining process. While this new approach and philosophy is demonstrated here on specific types of surface data, it has application to problems related to the analysis of any digital signals, images, surfaces and volumes.

Key words: digital data, surface, differential geometry, Dupin's indicatrix

INTRODUCTION

Geophysical analysis is often dealing with curves, surfaces and volumes in many subdomains.

There are two increasingly important aspects that need to be considered for the processing of these data:

- Dramatic increase in the spatial resolution as well as increase of amount of data.
- While the processing of these large datasets is certainly challenging, the automatic and unbiased extraction of accurate and meaningful attributes into query friendly databases will become essential.

When the digital world became a reality more than 30 years ago, who could have imagined the levels of analysis that would be applied to this digital data. However, the current methods used on almost all digital, including geophysical, data makes the direct and accurate application of various established mathematical theories impossible.

After more than 20 years of basic research in Numerical mathematics and Digital Geometry, new technology has been developed. GeoProxima was established in order to commercialise this breakthrough technology for digital data analysis into the field of geoscience.

METHODS AND RESULTS

Differential geometry is a mathematical discipline that combines the techniques of differential calculus and integral calculus, linear algebra and multi-linear algebra, to study problems in geometry. The differential geometry of surfaces captures many of the key ideas and techniques characteristic of this field. Carl Friedrich Gauss was a key contributor to the early stage of differential geometry development. He identified curvature as an intrinsic property which can change across a surface. Gauss and other important mathematicians developed theories describing the way of studying curves, surfaces and spaces. These theories are essential for many practical applications including geoscience.

Dupin's indicatrix

An important method for characterizing local shape of a continuous surface is Dupin's indicatrix (Dupin, 1813). It is a conic section curve describing all normal curvatures within an infinitely small neighborhood of any regular point of the surface. A continuous surface allows calculation of this indicatrix for any given point and thus there is an infinite number of Dupin's indicatrices for such surface.

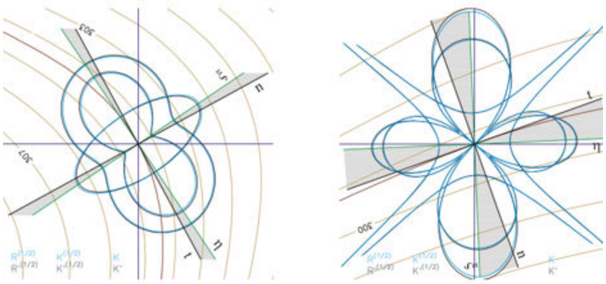

Figure 1. (KRCHO 2001) Shape of the Dupin indicatrix. Left side of the picture is showing elliptic Dupin indicatrix with Hyperbolic shown on the right side. Principal directions are shown as well as directions of tangent to the slope line and tangent to the contour line. Curves described by square roots of all normal curvatures themselves are shown as well.

Differentiability and interpolation issue in digital space

All formulas and notions of Differential geometry are valid for a regular function which is continuous and differentiable. In the past Mathematicians have observed that it is very difficult (or perhaps impossible) to reach practically acceptable estimates of partial derivatives of the first and second orders of the function using digital data. This problem also remained unsolved for the estimation of curvature. Due to digitization and the noise effects, it represents an ill-posed problem (and subsequent non-unique solution) because arbitrarily small permutations in the input can cause arbitrarily large permutations in the output (Figure 2).

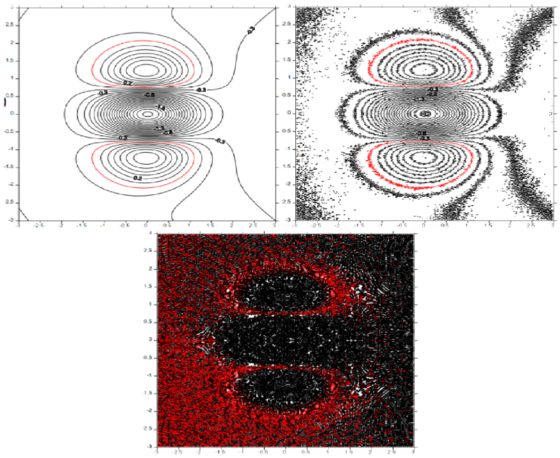

Figure 2. This group of contour maps demonstrates the effect of reduced precision on the accuracy of numerical derivatives.

The input was a mathematical function where a second order derivative was determined numerically with more than ten decimal place precision (Figure 2 upper left). As the computational precision becomes less than ten (10) decimal places, more noise is noticeable in the result. In this example using less than six (6) decimal places precision (upper right) produces a result that is compromised by noise introduced by the calculations. Unfortunately, this computational noise is often interpreted as signal which contains useful information. Note when only four (4) decimal precision is used the result is mostly noise! (Figure 2 bottom picture). Moreover, with real datasets, the presence of noise is also detrimental to the quality of the result. This same issue has effected historical calculations of surface curvatures. The issue with interpolation is that there are infinite amounts of possible functions which can be applied on discrete data (Figure 3). This yields both non-unique and unstable solutions. Moreover, many existing techniques require reduction of input data density in order to perform calculations. This leads to an information paradox; more information requires greater reduction. In practice that means, portions of that expensive acquired data has to be discarded.

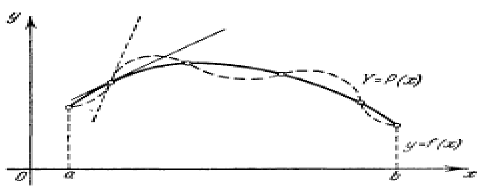

Figure 3. (DEMIDOVICH, MARON, 1981) Two curves are fit the same discrete points, however, their properties are completely different. Increasing the density of discrete points would only exacerbate this issue of interpolation.

New technology developed by GeoProxima

Despite enormous efforts in mathematics, computer science and the development of various methods for estimations of surface derivatives and curvatures in digital space, only the new methodology presented here is able to satisfy the criteria of stability, convergence, uniformity and automation of whole processing at the same time. This in part represents an aspect of this mathematical breakthrough. Our approach addresses the fundamental problems which have plagued surface analysis. We've developed an application based on our mathematical theory backed by rigorous mathematical proofs (thereby avoiding the ill-posed problem of numerical differentiation and interpolation) by implementing topological methods respecting existing theory in differential geometry. These calculations are well defined, stable and are well suited for automation. Probably the most important characteristic is the convergence of results to the analytically defined solution as grid density is increased. The most significant step of the presented approach to digital data analysis is the proper calculation of all geometric properties of a digital curve. We achieve this using a unique approximation technique in an automated manner which has both high efficiency in noise filtering and produces highly stable results. Application of the technology greatly compresses waveform data, eliminates noise on any part of a waveform and preserves objects enabling their automatic extraction (Figure 4).

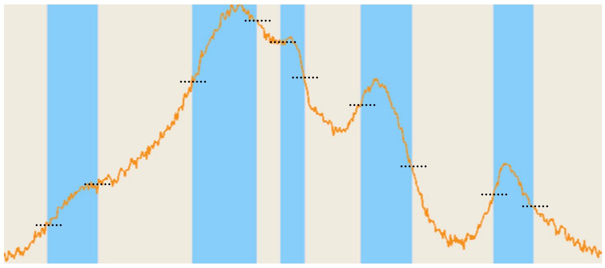

Figure 4. Example shows result of result of object identification on a noisy signal curve using our technique. Blue vertical stripes are marking localised convex objects on a curve, gray stripes are concave. Boundaries of objects (black dotted lines) are not on equiconstant levels. These boundaries cannot be accurately identified using threshold techniques or the application of gradational colors.

In addition to the calculation of properties of a curve in digital space, the same approach can be applied to the problem of surface analysis as is defined in the theories of Differential geometry (Krcho, 2001). For every point of the digital surface all Normal Curvatures are calculated resulting in the construction of the Dupin indicatrix. This enables the interrogation of the shape of the digital surface in any direction. Thus enabling complete morphometric analysis and automatic creation of objects. Moreover, automatic vectorisation allows the transfer of specified geometric properties from raster to vector form and greatly reduces the volume of data. This is crucial for creation of databases in order to highly simplify and speed up any data mining process. To demonstrate the significance of this new approach our method and an existing and frequently used method are applied to a mathematical surface containing a known mathematically defined object (Figure 5). The example shown demonstrates how artefacts can dominate the results using traditional methods of analysis. In this case, since the input model is known, one of analysis methods (Roberts 2001, Woods 1996) is completely unsuited as the results are completely dominated by the high frequency coherent noise introduced by the analysis technique. Since we know real datasets contain noise, but we do not know apriori what objects are to be found in real datasets, why would we expect these algorithms to work any better on real data (without heavily smoothing the input data).

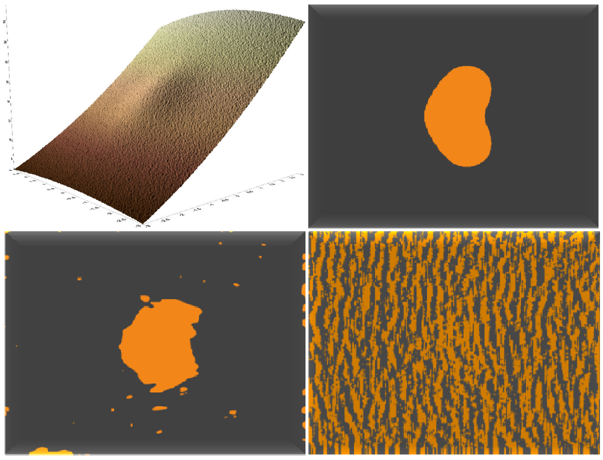

Figure 5. 3D view of an analytical function with added Gaussian noise (upper left). A known object of Convex Dip curvature is present (upper right) is revealed after automatic calculation of Dip curvature using GeoProxima (lower left). Whereas application of a commonly used technique fails (lower right).

Application in Geophysics

The GeoProxima approach allows automatic analysis of digital geodata automated object extraction and the creation of object geodatabases along with the necessary files for data exchange and interoperability. Therefore, there is no longer the need for intensive parameterisation and iterative data processing enabling the geoscientist to focus on the interpretation of results instead. Three examples of geophysical surface analysis provide insight on the applicability of this new technology. These are (1) Potential field data objects extraction, (2) Seismic surface fault identification, (3) Seafloor vent (Pockmark) delineation.

1. Potential fields data analysis example.

Images from a portion of a Gravity dataset (~200 Sq. Kms) collected over a paleo-caldera are shown (Figure 6) with different gradational palettes applied. In this example, the grayscale image gives an indication of a central caldera and other morphology. However, the colour palette provides a different and in this case, false impression. Moreover, gradational color palettes do not identify objects in the surface. Using the new approach to surface analysis described, objects of localized curvature were automatically extracted (Figure 7). The objects are colored by the mean Z value within each boundary demonstrating that the object boundaries cross equi-constant levels and cannot be located with contour lines.

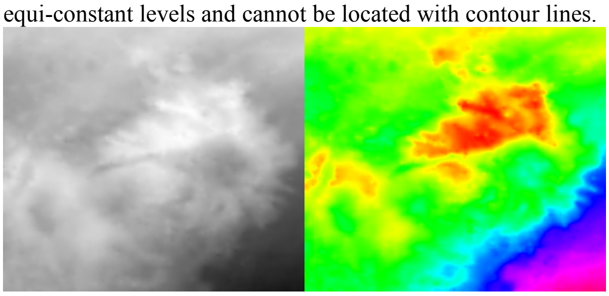

Figure 6. Potential field data. Gravity grayscale map of an eroded caldera (left). Gradational false colour palette applied to the same data (right) gives a false impression of the data.

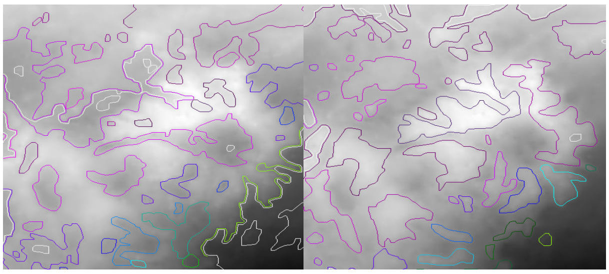

Figure 7. Objects of localised Concave (left) and Convex (right) were automatically extracted using this new approach.

2. Faults on seismic surface example.

A TWT surface extracted from a North Sea 3D seismic data volume (~300 Sq. Kms) (Figure 8 and 9) is first analysed with a commonly used surface or volume analysis techniques implementing windowed methods for surface analysis. Typically, results using larger windows have fewer artefacts at the expense of high frequency detail.

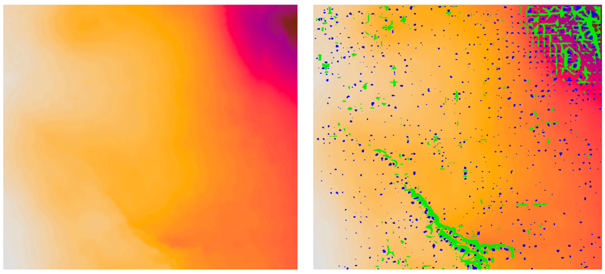

Figure 8. Extract from seismic surface in false colors (left). Result of curvature analysis (right) using existing techniques (using 7x7 operator). Convex (Green), Concave (Blue).

Analysis of the TWT surface (Figure 8 left) using a popular and frequently used technique (Roberts, 2001) shows results biased by calculation noise (Figure 8 right), making it difficult to discriminate between what is real and artefacts. Using the new method we describe, results after analysis of the same surface (Figure 9 left) shows a much clearer picture, without the artefacts. The concave and convex objects or the automatically vectorised version of the data provides an objective quantification of localised convex and concave features on the surface (Figure 9 right).

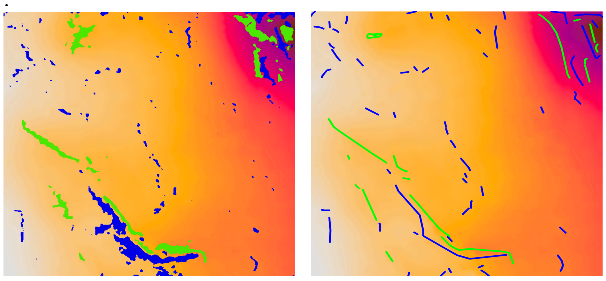

Figure 9. Applying GeoProxima approach to same surface clearly highlights localised trends of convex (anticlinal) and concave objects (synclinal). The image on the right shows how these objects can be automatically vectorised.

3. Pockmarks database creation example.

Pockmarks are conical depressions which are observed on the seafloor and form as fluids from deeper in the section vent into the ocean. These features can provide insights into geohazards (e.g. seafloor stability), fluid migration pathways and the presence of hydrocarbon accumulations at depth (Hovland and Judd, 2007). The Two Way Time (TWT) rendered surface of the Seafloor (Figure 10 top) shows dramatic changes in depth over a shelf break. Several submarine canyons can be seen along with a slump area. The surface was processed for automatic object extraction (pockmarks) creating a geodatabase of geometric and morphometric properties for each feature. Single or multi-level queries made to this database can create maps highlighting different aspects of these pockmarks {e.g. density probability (shown), aspect ratio, size, orientation etc.,). Moreover, this spatial information can be exported as shape files into GIS or modelling applications for the creation of local or regional multi-scale geomodels. In this example, providing pockmark morphology maps showing the distribution of seafloor venting over the field, survey or basin.

CONCLUSIONS

A fresh new technology for digital data analysis is presented which does not use existing methods such as: Operators (Canny, Prewitt, Sobel, Kirsch); Krigging, Splines, Inverse Distance Weighting; Geostatistics, Transform (ie. FFT, DCT, etc.; Wavelets; Gaussian decomposition; Worms; Derivatives (1st, 2nd, etc.,). The approach is suitable for wide range of geophysical data processing including signals, seismic surfaces, potential fields data, bathymetric measurements. All processing is performed automatically allowing for the creation of queryable geodatabases. With automated genetic analysis techniques extracting thousands of high quality surfaces from 3D seismic datasets (Dirstein, 2011) which collectively cover hundreds of thousands of square kilometres, this objective analysis technique is particularly suited for the multi-scale analysis of these and other large high resolution (and high density) datasets. While our eureka moment took more than a decade to achieve, the benefits of this approach applied to many different types of data is happening in a more timely manner.

REFERENCES

DEMIDOVICH, B. P., MARON, I. A. 1981. Computational mathematics. Moscow, MIR Publishers.

DIRSTEIN, J.K. and G.N. FALLON., 2011. Automated Interpretation of 3D Seismic Data Using Genetic Algorithms, ASEG Preview v201 no. 151 pg 30-37.

DUPIN, C,. Developments in Geometry 1813.

GAUSS, C. F. 1827. (General investigation into curved surfaces).

HOVLAND, M. and JUDD, A, 2007. Seabed Fluid Flow, The Impact on Geology, Biology and the Marine Environment.

KRCHO, J. 2001. Modelling of Georelief and its geometrical structure using DTM: Positional and Numerical accuracy. Bratislava: Q111, 2001.

ROBERTS, A., 2001, Curvature attributes and their application to 3D interpreted horizons. FBreak, 19(2) p. 85-100.

ŠALÁT, T. et al. 1981. Malá encyklopédia matematiky. Bratislava: Obzor, 1981

WOODS,1996. The geomorphological characterisation of digital elevations models PhD Thesis, U of Leicester, UK.

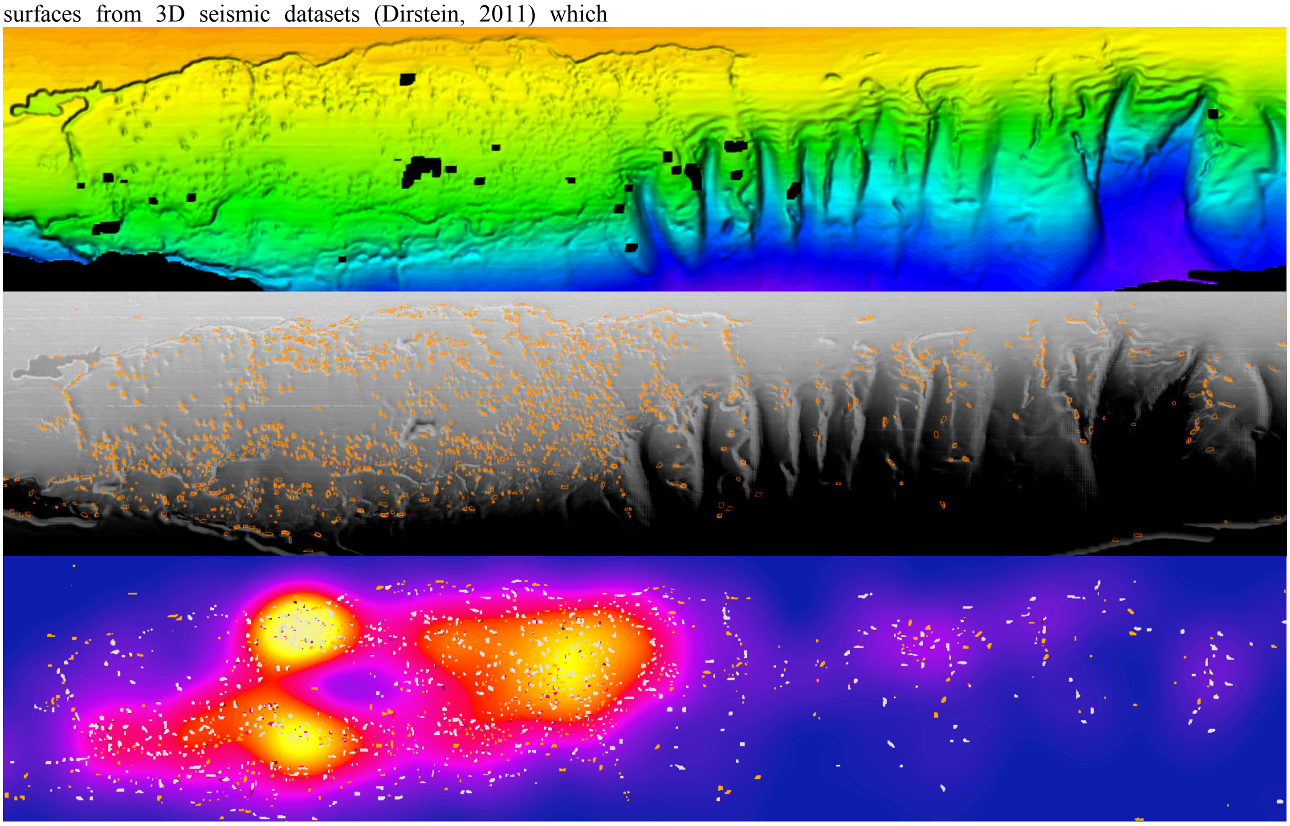

Figure 10. Seismic surface analysis example (TWT surface from portion a 3D seismic dataset (~ 500 Sq. Km). False colored map of TWT with relief shading (top). Orange objects overlayed on a grayscale TWT map as a result of pockmarks detection by GeoProxima (middle). Calculated pockmarks density probability with overlayed pockmark objects (bottom).